Optimizing the configuration of X-by-Wire networks using word combinatorics

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TTP/C – Time Triggered Protocol

- Designed at T.U. Vienna + TTTech
- TTP/C main technical characteristics:
  - Determinism
  - Fault-Tolerance
  - Composability
  - Support of mode changes

⇒ A good candidate for X-By-Wire ..
X-by-Wire

- Hydraulic and mechanical connection are replaced by networks and actuators

Why?
- Decrease of weight and cost
- Safety: intrusion of the steering column in the cockpit
- New functions: variable demultiplication - crash avoidance
- Less pollution (brake / transmission liquid)
- ...

X-by-wire: an example
A TTP/C Cluster

- Medium Access Control: TDMA
- Redundant transmission support
- Data rate: 500kbit/s, 1Mbit/s, 2Mbit/s, 5Mbit/s, 25Mbit/s
- Topology: bus or star

TDMA – Time division Multiplexed Access

- Slot: time window given to a station for a transmission
- TDMA Round: sequence of slots s.t. each station transmits exactly once
- Cluster Cycle: sequence of the TDMA rounds
TTP/C: Implications of the MAC protocol

Bounded response times and « heartbeats » but:
- loss of bandwidth
- need of powerful CPU’s
- maximum timing contraint:
  - If a station sends a single information, the refresh cannot be more frequent than the length of a round
  - If a station sends several informations, the refresh cannot be more frequent than 2x the length of a round

Ex: 5ms time constraint - 500kbit/s network with 200 bits per frames - at most 12 frames (6 FTUs of two nodes) or 6 frames if the station sends 2 distinct informations

FTU: Fault Tolerant Unit

- FTU = set of stations that act identically

- Replica = a frame sent by a node of the FTU
FTU: which protection?

- Protection against:
  - disappearance of a station (crash, disconnection..)
  - corrupted frames (EMI)
  - sensors or computation errors
  - ...

- Under the assumption of a single failure (TTP/C fault-hypothesis):
  - A dual redundancy ensures a protection in « the temporal domain »
  - A triple redundancy ensures in addition a protection in « the value domain »

- Problem: history-state

Goal of the study: maximize the robustness against transmission errors

- Transmission errors are usually highly correlated

Where to place the replicas?
Application model

- $T_a$: production cycle of the data sent by the stations of the FTU

Assumptions:
- no synchronization between production and transmission (round)
- production cycle is a multiple of the length of a round

Objective w.r.t. fail-silence

- A node is « fail-silent » if one can safely consume its data when the frame carrying the data is syntactically correct

- Stations are fail-silent: « minimize $P_{\text{all}}$ : the probability that all frames of the FTU sent during a production cycle will be corrupted »

- Stations are not fail-silent: « minimize $P_{\text{one}}$ : the probability that at least one frame of the FTU will be corrupted »
Assumptions on the error model

- Each bit transmitted during an EMI will be corrupted with probability $\pi$
- If a perturbation overlaps a whole slot, the corresponding frame is corrupted with probability $1$
- Starting times of the EMI bursts are independent and uniformly distributed over time
- The distribution of the size of the bursts is arbitrary

Objective 1: Minimize $P_{one}$
Majorization - Schur-Convexity

- A vector $u = (u_1, ..., u_n)$ majorizes $v = (v_1, ..., v_n)$ if:
  \[ \sum_{i=1}^{n} u_i = \sum_{i=1}^{n} v_i \text{ and } \sum_{i=1}^{k} u_{[i]} \leq \sum_{i=1}^{k} v_{[i]}, \quad k \leq n \]
  with $(u_{[i]}, ..., u_{[n]})$ permutation of $u$ s.t. $u_{[i]} \leq ... \leq u_{[n]}$.

- Example: $(1, 3, 5, 10) > (2, 4, 4, 9)$

- A function $f : \mathbb{R}^n \rightarrow \mathbb{R}$ is
  - Schur-convex if $u > v \rightarrow f(u) \geq f(v)$
  - Schur-concave if $u > v \rightarrow f(u) \leq f(v)$

Minimizing $P_{one}$

- $I(x)$ is the vector of the distance between replicas during the length of a round (sorted in ascending order).

- Example: $I(x) = (1, 1, 2)$

- Example: $I(x) = (0, 0, 4)$
Minimizing $P_{\text{one}}$

**Theorem:** the best allocation for $P_{\text{one}}$ is to group together all replicas (denoted allocation $g$)

**Arguments:**
- $P_{\text{one}}$ is shur-concave: $I(x') > I(x) \Rightarrow P_{\text{one}}(x') \leq P_{\text{one}}(x)$
- $I(g)$ is maximum for the majorization (equal to $(0, 0, ..., S - k)$ with $k$ the number of replicas of the FTU and $S$ the number of slots per round)

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Minimize $P_{\text{one}}$

- **Idea of the proof (step 1):** the farther the beginning of an error burst from a replica, the less likely the replica becomes corrupted. « Non-grouped » allocations have more areas close to replicas
Minimize $P_{one}$

- Validity of the result:
  - Arbitrary $\pi$ value and burst size distribution
  - Production period multiple of the round length
  - for all TDMA networks

- Combined minimization of $P_{one}$ for all FTU’s is possible

- Robustness improvement: against a random allocation, the number of lost data is reduced from 15 to 20% on average

Objective 2: Minimize $P_{all}$

TTP/C case
**TTP/C : the majority rule**

- **Cliques:** sets of stations that disagree on the state of the network
- **Principle:** to avoid cliques, stations in the minority disconnect (« freeze »)
- **Mechanism:** before sending, a station checks that in the last round (S slots), the number of correct messages is greater than the number of incorrect messages, otherwise it disconnects

\[ a_i \quad a_{i-1} \quad \text{S slots in a round} \]

- If a station « freezes » due to transmission errors, the others follow one by one...

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**TTP/C : minimize \( P_{\text{all}} \)**

**Algorithm:**

1) for each FTU \( i \) with \( C_i \) slots, push \( \lceil C_i / 2 \rceil \) slots in the smallest stack and \( \lfloor C_i / 2 \rfloor \) in the largest stack
2) concatenate the two stacks

**Ex:** FTU A: 3 replicas – FTU B: 2 replicas – FTU C: 4 replicas

![](image.png)
Theorem: the « 2-stacks » algorithm is optimal under TTP/C

Arguments:
Case 1) a perturbation for each replica: identical ∀ allocation
Case 2) a perturbation can corrupt several replicas with a probability decreasing in the distance between the replicas. A burst of more than \[ \left\lfloor \frac{S}{2} \right\rfloor \] slots freezes the system, now the algorithm ensures a distance of \[ \left\lfloor \frac{S}{2} \right\rfloor \] slots

Corollary: it is useless to have more than 2 replicas per FTU if the probability to have more than one perturbation in the same round is sufficiently low

Objective 2: Minimize \( P_{all} \)

TDMA case
Balanced words

- A « balanced » word (or Sturmian word) is a binary sequence \( \{u_n\}_{n \in \mathbb{N}} \) s.t.:
  \[
  \forall k, n, m \in \mathbb{N}, \sum_{i=n}^{n+k} u_i - \sum_{j=m}^{m+k} u_j \leq 1
  \]

- Balanced words are computed using bracket sequences:
  \[
  u_n = \left\lfloor \frac{n}{a} \right\rfloor - \left\lfloor (n-1) \frac{a}{b} \right\rfloor
  \]
  where \( a/b \) is the rate of the word (nb of 1 / nb of 0)

- Example: balanced word of rate 3/8
  \((0,0,1,0,0,1,0,1)\)

Multimodular functions

- Multimodularity [Hajek]: counterpart of convexity for discrete functions \( f : \mathbb{Z}^m \rightarrow \mathbb{R} \)

Definition: Let \( F \) be a set of \( m+1 \)-vectors that sum to 0, a function \( f : \mathbb{Z}^m \rightarrow \mathbb{R} \) is \( F \)-multimodular if
  \[
  f(x + v) + f(x + w) \geq f(x) + f(x + v + w)
  \]
  \( x \in \mathbb{Z}^m \) et \( v, w \in F \), \( v \neq w \)

- Example: \( x = (0,1,0,...,1,1,0) \) is a control sequence, \( f \) a cost function and \( v \) an elementary operation moving a client to the left
  \[
  v = (0,...,0,1,-1,0,...,0)
  \]
Optimisation and multimodular function

- Global left shift operator: \( s_i(x) \)
  
  ex: \( s_2((0,1,0,1,1,0)) = (0,1,1,0,0,1) \)

**Theorem [Altman,Gaujal,Hordijk 97]**: If \( f \) is multimodular then \( G(x) = 1/m \sum_{i=1}^{m} f(s_i(x)) \) (shift-invariant version of \( f \)) is minimum (among all admissible sequences) if \( x \) is a balanced sequence.

Theorem: If the size of the bursts is exponentially distributed then \( P_{all} \) is multimodular. Moreover, \( P_{all} \) is equal to its shift invariant version thus \( P_{all} \) is minimum for a balanced sequence.

Optimal algorithm: a single FTU

- FTU with \( C \) replicas of size \( h \) bits in a round of total size \( R \) bits
  - compute \( v_i \) balanced word of rate \( C/(R-C(h-1)) \)
  - \( x \) is the round initially empty
  - If \( v_i = 1 \) then \( x := x + 1 \ldots 1 \) (\( h '1' \) concatenated)
  - If \( v_i = 0 \) then \( x := x + 0 \)

Ex: FTU: 3 replicas of cardinality 3 in a round of size 14

\( v_i = (0,0,1,0,0,1,0,1) \) with rate 3/8

\( x = (0,0,1,1,1,0,0,1,1,1,0,1,1,1) \) (not balanced word with rate 9/1).
Optimal algorithm: several FTUs

- Problem: allocation conflicts
  Ex: FTU A: 3 replicas – FTU B: 2 replicas – FTU C: 1 replica
  \[ x_A = (0,1,0,1,0,1) \text{ rate } 3/6 \]
  \[ x_B = (0,0,1,0,1,0) \text{ rate } 2/6 \]
  \[ x_C = (0,0,0,0,0,1) \text{ rate } 1/6 \]

- An optimal allocation is still possible if
  - the number of cardinalities is a power of 2
  - all replicas have the same size

- Remark: a balanced sequence is minimum for the majorization order, it is thus the worst solution for Pone! Both objectives are contradictory

Heuristic: several FTUs

- The rate of emission: number of bits that FTU A must emit on average during each bit of a round:
  \[ d_A = C_A h_A / R \]

- At step i, one schedules the transmission of a frame of the FTU for which the number of due bits – number of already allocated bits is maximum
**Pall : Heuristic vs random allocation**

- Reduction of the number of lost messages w.r.t. a random allocation:

![Graph showing reduction of lost messages](image)

**Pall : Heuristic vs optimal**

- Increase of the number of lost messages w.r.t. to the optimal:

![Graph showing increase of lost messages](image)
Conclusion

Optimal and near optimal allocation policies for TDMA and TTP/C networks

- Choice of the locations of the slots have a strong influence on the robustness of the network
- The cost function plays a major role on the shape of the solution
- Hypothesis on the error model are crucial

Future work:
- Configurations made of fail-silent and non fail-silent nodes (minimizing $P_{one}$ and $P_{all}$ for different FTU’s)
- FlexRay protocol